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非対称に単項式のノルムを保存する関数環上の写像について
(MAPS BETWEEN UNIFORM ALGEBRAS WHICH PRESERVE THE NORMS OF MONOMIALS NON-SYMMETRICALLY)

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1. Introduction

Let $C(X)$ be the set of all complex-valued continuous functions on a compact Hausdorff space X and $\|f\|_\infty = \sup_{x\in X} |f(x)|$ the supremum norm on X for $f \in C(X)$. Then $C(X)$ is a Banach algebra with pointwise multiplication and the supremum norm. The subset A of $C(X)$ is said to be a uniform algebra on X if A is a closed subalgebra of $C(X)$ which separates the points of X and contains the constant functions. Let A and B be uniform algebras on compact Hausdorff spaces X and Y respectively. For $f \in A$, let $\sigma(f)$ be the spectrum of f. Recall that $f(X)$ is a subset of $\sigma(f)$ and $\|f\|_\infty$ equals the spectral radius of f.

Molnár [10] showed the following:

Theorem 1. (*Molnár* [10]) *If X is first-countable and T is a surjection from $C(X)$ onto itself with $\sigma(T(f)T(g)) = \sigma(fg)$ for all $f, g \in C(X)$, then $T/T(1)$ is an algebra isomorphism.*

Rao and Roy [11] extended this result (see also [2, 3, 6, 7]). Most recently, Hatori, Hino, Miura and Oka [4] generalized their results. In particular, they showed the following:

Theorem 2. (*Hatori, Hino, Miura and Oka* [4, Theorem 1.1]) *Let $\sigma_\pi(f) = \{f(x) : x \in X, |f(x)| = \|f\|_\infty\}$ for $f \in A$. If a surjection $T : A \to B$ satisfies $\sigma_\pi(T(f)^m T(g)^n) \subset \sigma_\pi(f^m g^n)$ for some fixed positive integers m, n and all $f, g \in A$, then there exists a real-algebra isomorphism $\widetilde{T}$ such that $\widetilde{T}(f)^d = (T(f)/T(1))^d$ for every $f \in A$, where d is the greatest common divisor of m and n.*

Hatori, Miura and Takagi [3, Corollary 7.5], and Luttman and Lambert [8] independently showed the following:

Theorem 3. (*Hatori, Miura and Takagi* [3, Corollary 7.5], *and Luttman and Lambert* [8]) *If a surjection $T : A \to B$ satisfies $\|T(f)T(g) - \alpha\|_\infty = \|fg - \alpha\|_\infty$ for some fixed non-zero complex number α and all $f, g \in A$, then $T/T(1)$ is a real-algebra isomorphism.*

Note that, for some fixed complex number α and $f, g \in A$, $\sigma(f) = \sigma(g)$ if and only if $\sigma(f - \alpha) = \sigma(g - \alpha)$, which implies $\|f - \alpha\|_\infty = \|g - \alpha\|_\infty$. Hence their result is a generalization of Theorem 1 (see also [5, 6, 9]). We denote by A^{-1} the set of invertible

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elements of A. Let $\hat{f}$ be the Gelfand transform of $f \in A$, M_A the maximal ideal space of A and $\bar{\cdot}$ the complex conjugate. Our main result is the following:

Theorem 4. [12, Theorem 1.2] *Let m, n be positive integers and α a non-zero complex number. Suppose that S_A, S_B are subsets of A, B that contain A^{-1}, B^{-1} respectively. If $T : S_A \longrightarrow S_B$ is a surjection such that*

$$\|T(f)^m T(g)^n - \alpha\|_\infty = \|f^m g^n - \alpha\|_\infty \tag{1}$$

for all $f, g \in S_A$, then there exist a real-algebra isomorphism $\widetilde{T} : A \longrightarrow B$, a clopen subset $\mathcal{K}$ of M_B and a homeomorphism $\Phi : M_B \longrightarrow M_A$ such that

$$\widehat{\widetilde{T}(f)} = \begin{cases} \hat{f} \circ \Phi & \text{on } \mathcal{K} \\ \overline{\hat{f} \circ \Phi} & \text{on } M_B \setminus \mathcal{K} \end{cases}$$

for every $f \in A$ and $\widetilde{T}(f)^d = (T(f)/T(1))^d$ for every $f \in S_A$, where d is the greatest common divisor of m and n.

2. A proof of Main result

We denote by $\exp A$ the range of the exponential map on A. Let $\sigma_\pi(f) = \{f(x) : x \in X, |f(x)| = \|f\|_\infty\}$ for $f \in A$ and $P_{\exp A}(x) = \{u \in \exp A : \sigma_\pi(u) = \{1\}, u(x) = 1\}$ for $x \in X$. If $\sigma_\pi(p) = 1$ for $p \in A$, then p is called a peaking function of A. For a peaking function p, the set of points on which p takes the value 1 is called the peak set of p. A point $x \in X$ is called a weak peak point of A if the set $\{x\}$ equals the intersection of a family of peak sets of A. The set $\mathrm{Ch}(A)$ of all weak peak points of A coincides with the Choquet boundary of A. It is known that $\mathrm{Ch}(A)$ is a boundary for A. In order to prove the main theorem, we will need Lemma 5, 6 and Proposition 7.

Lemma 5. (*cf.* [4, Proposition 2.2]. *See also* [1, 2, 3, 5, 6, 8, 9, 11].) *Let $v \in A^{-1}$ and $x_0 \in \mathrm{Ch}(A)$. If F is a closed subset in X with $x_0 \notin F$, there exists a $u \in P_{\exp A}(x_0)$ such that $\sigma_\pi(uv) = \{v(x_0)\}$ and $|uv| < |v(x_0)|$ on F.*

Lemma 6. (*cf.* [8, Lemma 2.1].) *Let $f_1, f_2 \in A$. If $\|f_1 g - 1\|_\infty = \|f_2 g - 1\|_\infty$ for all $g \in \exp A$, then $f_1 = f_2$.*

Proposition 7. [12, Proposition 2.6 and 3.2] *Suppose that A_0, B_0 are subgroups of A^{-1}, B^{-1} that contain $\exp A, \exp B$ respectively. If $S : A_0 \longrightarrow B_0$ is a surjection such that $S(1) = 1$ and*

$$\|S(f)S(g)^{-1} - 1\|_\infty = \|fg^{-1} - 1\|_\infty \tag{2}$$

for all $f, g \in A_0$, then there exist a real-algebra isomorphism $\widetilde{T} : A \longrightarrow B$, a clopen subset $\mathcal{K}$ of M_B and a homeomorphism $\Phi : M_B \longrightarrow M_A$ such that

$$\widehat{\widetilde{T}(f)} = \begin{cases} \hat{f} \circ \Phi & \text{on } \mathcal{K} \\ \overline{\hat{f} \circ \Phi} & \text{on } M_B \setminus \mathcal{K} \end{cases}$$

for every $f \in A$ and $\widetilde{T}(f) = S(f)$ for every $f \in A_0$.

Proof. We begin by showing that there exists a homeomorphism ϕ from $\mathrm{Ch}(B)$ onto $\mathrm{Ch}(A)$ such that

$$|S(f)(y)| = |f(\phi(y))| \tag{3}$$

for every $f \in A_0$ and $y \in \mathrm{Ch}(B)$ (cf. [3, 4, 5, 9]). For $y \in \mathrm{Ch}(B)$, let

$$W_y = \{f \in B_0 : |f(t)| = 1 = \|f\|_\infty\}.$$

Then, $P_{\exp B}(y)$ is a subset of W_y. For every $y \in \mathrm{Ch}(B)$, the set $\cap_{f\in S^{-1}(W_y)}|f|^{-1}(\{1\})$ is a singleton that belongs to $\mathrm{Ch}(A)$. If $\phi(y)$ is the single element, i.e.

$$\{\phi(y)\} = \cap_{f\in S^{-1}(W_y)}|f|^{-1}(\{1\}),$$

we can define the mapping $\phi : y \mapsto \phi(y)$ from $\mathrm{Ch}(B)$ into $\mathrm{Ch}(A)$. Then $\phi : \mathrm{Ch}(B) \to \mathrm{Ch}(A)$ is bijective and satisfies (3). This implies the continuities of ϕ and ϕ^{-1}.

Let $y \in \mathrm{Ch}(B)$ and $S^1 = \{z; \text{a complex number with } |z| = 1\}$. We will show that

$$S(f) = \begin{cases} f \circ \phi & \text{if } y \in K \\ \overline{f \circ \phi} & \text{if } y \in \mathrm{Ch}(B) \setminus K \end{cases} \tag{4}$$

for every $f \in A_0$ (cf. [3, 5, 8, 9]). For every $\beta \in S^1$ and $\mathrm{u} \in P_{\exp B}(y)$, there exists a $u \in A_0$ such that $S(u) = S(\beta)\mathrm{u}$. By (3), we have $|u(\phi(y))| = 1$. We also have $\|S(\beta)\mathrm{u}/S(-u(\phi(y)))\|_\infty = 1$. Equation (2) shows that

$$\left\|\frac{S(\beta)\mathrm{u}}{S(-u(\phi(y)))} - 1\right\|_\infty = \left\|-\frac{u}{u(\phi(y))} - 1\right\|_\infty = 2,$$

which implies that there exists a $y' \in \mathrm{Ch}(B)$ with $S(-u(\phi(y)))(y') = -S(\beta)(y')\mathrm{u}(y')$. Since $|\mathrm{u}(y')| = 1$ and $\mathrm{u} \in P_{\exp B}(y)$, we obtain $\mathrm{u}(y') = 1$, so by (3),

$$2 = \left|\frac{S(-u(\phi(y)))(y')}{S(\beta)(y')} - 1\right| \le \left\|\frac{S(-u(\phi(y)))}{S(\beta)} - 1\right\|_\infty \le \left\|\frac{S(-u(\phi(y)))}{S(\beta)}\right\|_\infty + 1 = 2.$$

Thus, by (2), $|-u(\phi(y))\beta^{-1} - 1| = 2$, which shows that

$$u(\phi(y)) = \beta. \tag{5}$$

Since, by (2) and (3), $\|S(\beta)\mathrm{u}S(-\beta)^{-1} - 1\|_\infty = \|S(\beta)\mathrm{u}S(-\beta)^{-1}\|_\infty + 1 = 2$, there exists a $y_\beta \in \mathrm{Ch}(B)$ such that $S(-\beta)(y_\beta) = -S(\beta)(y_\beta)\mathrm{u}(y_\beta)$. Notice that $|\mathrm{u}(y_\beta)| = 1$ and $\mathrm{u} \in P_{\exp B}(y)$, which implies that

$$\mathrm{u}(y_\beta) = 1 \text{ and } S(-\beta)(y_\beta) = -S(\beta)(y_\beta). \tag{6}$$

Applying Lemma 5 for $S(1)^{-1} \in B^{-1}$ and equation (6) for $\beta = 1$, we obtain $S(-1)(y) = -1$ for every $y \in \mathrm{Ch}(A)$. Thus, by (2), we have $\|S(\beta) - 1\|_\infty = |\beta - 1|$ and $\|S(\beta) + 1\|_\infty = |\beta + 1|$ for every $\beta \in S^1$. Since $|S(\beta)| = 1$ on $\mathrm{Ch}(B)$, we obtain $S(\beta)(\mathrm{Ch}(B)) = \{\beta, \overline{\beta}\}$ for every $\beta \in S^1$. Define

$$K = \{y \in \mathrm{Ch}(B) : S(i)(y) = i\}.$$

Then K is a clopen subset of $\mathrm{Ch}(B)$ and the closures in Y of K and $\mathrm{Ch}(B) \setminus K$ are disjoint. Let F_0 be the closure in Y of K or $\mathrm{Ch}(B) \setminus K$ with $y \notin F_0$. Applying Lemma 5 for $S(i)^{-1} \in B^{-1}$, $F_0 \subset Y$ and equation (6) for $\beta = i$, we obtain $S(-i)(y) = -S(i)(y)$ for every $y \in \mathrm{Ch}(B)$. Together with equations (2) and (3), this shows that $\|S(\beta) - S(i)\|_\infty = |\beta - i|$ and $\|S(\beta) + S(i)\|_\infty = |\beta + i|$. Hence,

$$S(\beta)(y) = \begin{cases} \beta & \text{if } y \in K \\ \overline{\beta} & \text{if } y \in \mathrm{Ch}(B) \setminus K \end{cases} \tag{7}$$

for every $\beta \in S^1$. Given $f \in A_0$, set $\beta_0 = -f(\phi(y))|S(f)(y)|^{-1}$. Then $\beta_0 \in S^1$. By Lemma 5, there exists a $\mathrm{u}_0 \in P_{\exp B}(y)$ such that

$$\sigma_\pi(\mathrm{u}_0 S(f)^{-1}) = \{S(f)(y)^{-1}\} \text{ and } |\mathrm{u}_0 S(f)^{-1}| < |S(f)(y)|^{-1} \text{ on } \mathrm{Ch}(B) \setminus F_0.$$

Applying (5) for $\beta = \beta_0$, there exists a $u_0 \in A_0$ such that $S(u_0) = S(\beta)\mathrm{u}_0$ and $u_0(\phi(y)) = \beta_0$. This shows that, by (2),

$$\left\| \frac{S(\beta_0)\mathrm{u}_0}{S(f)} - 1 \right\|_\infty = \left\| \frac{u_0}{f} - 1 \right\|_\infty \geq \left| \frac{\beta_0}{f(\phi(y))} - 1 \right| = |S(f)(y)|^{-1} + 1.$$

By (3), we have $\|S(\beta_0)\mathrm{u}_0 S(f)^{-1}\|_\infty = |S(f)(y)|^{-1}$, that is

$$\left\|S(\beta_0)\mathrm{u}_0 S(f)^{-1} - 1\right\|_\infty = |S(f)(y)|^{-1} + 1.$$

Hence there exists a $y_0 \in \mathrm{Ch}(B)$ such that

$$(S(\beta_0)\mathrm{u}S(f)^{-1})(y_0) = -|S(f)(y)|^{-1}.$$

The hypotheses of u_0 and equation (7) imply that

$$(\mathrm{u}_0 S(f)^{-1})(y_0) = S(f)(y)^{-1} \text{ and } S(\beta_0)(y_0) = S(\beta_0)(y),$$

which shows (4).

We will show that there exists a real-algebra isomorphism $\widetilde{T} : A \to B$ (cf. [3, 4, 9]). For each $f \in A$, there exist a complex number λ_0 and an $f_0 \in A_0$ such that the imaginary part of λ_0 is not zero, the real part of f_0 is positive and $f = f_0 + \lambda_0$. Notice that $f_0 \in \exp A$. Thus $f_0, \lambda_0 \in A_0$. Define a map $\widetilde{T}$ on A by

$$\widetilde{T}(f) = S(f_0) + S(\lambda_0).$$

Then, by (4), $\widetilde{T}$ is a real-algebra isomorphism such that

$$\text{(8)} \qquad \widetilde{T}(f) = \begin{cases} f \circ \phi & \text{on } K \\ \overline{f \circ \phi} & \text{on } \mathrm{Ch}(B) \setminus K \end{cases}$$

for every $f \in A$ and $\widetilde{T} = S$ on A_0.

Finally, we will construct a homeomorphism Φ from M_B onto M_A (cf. [8, The proof of Theorem 2.1]). By (8), we obtain $\widehat{\widetilde{T}(i)}(M_B) \subset \{i, -i\}$. Define a subset $\mathcal{K}$ of M_B by

$$\mathcal{K} = \{y \in M_B : \widehat{\widetilde{T}(i)}(y) = i\}.$$

Then $\mathcal{K}$ is a clopen subset of M_B with $\mathrm{Ch}(B) \cap \mathcal{K} = K$. Let $e = (\widetilde{T}(i) + i)/(2i)$, then e is an idempotent such that

$$\text{(9)} \qquad \hat{e} = \begin{cases} 1 & \text{on } \mathcal{K} \\ 0 & \text{on } M_B \setminus \mathcal{K} \end{cases}.$$

For $y \in M_B$, let $\Phi(y)$ be defined as

$$\Phi(y)(f) = \widehat{\widetilde{T}(f)}(y)\hat{e}(y) + \overline{\widehat{\widetilde{T}(f)}}(y)(1 - \hat{e})(y)$$

for every $f \in A$. Then, by (8) and (9), the mapping $\Phi : y \mapsto \Phi(y)$ is a homeomorphism from M_B onto M_A. By the definition of Φ and equation (9), we obtain the conclusion. $\square$

Here we prove Theorem 4, stated in the first section. Below we make use of subsets of A^{-1} defined as follows: Let k, l be positive integers and $\mathcal{X}$ a subset of a uniform algebra A. Define a subset $(\mathcal{X})_l^k$ of A by

$$(\mathcal{X})_l^k = \{f \in \mathcal{X} : \text{there exists an } f' \in \mathcal{X} \text{ with } f^k(f')^l = 1\}.$$

Then $(\mathcal{X})_l^k$ is a subset of $(A^{-1})_l^k$.

Proof of Theorem 4. Recall that S_A, S_B are subsets of A, B that contain A^{-1}, B^{-1} respectively and $T : S_A \to S_B$ is a surjection such that

$$\|T(f)^m T(g)^n - \alpha\|_\infty = \|f^m g^n - \alpha\|_\infty \tag{1}$$

for all $f, g \in S_A$. By a simple calculation, we obtain $(S_A)^m_n = (A^{-1})^m_n$ and $(S_B)^m_n = (B^{-1})^m_n$, since S_A, S_B contain A^{-1}, B^{-1} respectively. We will show that $T((A^{-1})^m_n) = (B^{-1})^m_n$. Suppose that ν_α is a complex number with $(\nu_\alpha)^n = \alpha$. For every $g \in (A^{-1})^m_n$, let $g' \in A^{-1}$ with $g^m (g')^n = 1$. Since, by (1),

$$\|T(g)^m T(\nu_\alpha g')^n - \alpha\|_\infty = \|g^m (\nu_\alpha g')^n - \alpha\|_\infty = \|g^m \alpha (g')^n - \alpha\|_\infty = 0,$$

we obtain

$$T(g)^m T(\nu_\alpha g')^n = \alpha.$$

This shows that $T(g)^m ({\nu_\alpha}^{-1} T(\nu_\alpha g'))^n = 1$, that is $T(g) \in (B^{-1})^m_n$. Together with the surjectivity of $\widetilde{T}$, similar arguments show the opposite inclusion. Consequently, $T((A^{-1})^m_n) = (B^{-1})^m_n$. Furthermore, we have

$$\begin{aligned} \left\| \frac{T(f)^m}{T(g)^m} - 1 \right\|_\infty &= \frac{1}{|\alpha|} \|T(f)^m T(\nu_\alpha g')^n - \alpha\|_\infty \\ &= \frac{1}{|\alpha|} \|f^m (\nu_\alpha g')^n - \alpha\|_\infty = \left\| \frac{f^m}{g^m} - 1 \right\|_\infty \end{aligned} \tag{10}$$

for every $f \in S_A$ and $g \in (A^{-1})^m_n$. Define a map T_m on $((A^{-1})^m_n)^m = \{f^m ; f \in (A^{-1})^m_n\}$ by

$$T_m(f^m) = T(f)^m / T(1)^m$$

for $f^m \in ((A^{-1})^m_n)^m$. Then, by (10), T_m is well-defined in the sense that $T(f)^m = T(g)^m$ for every $f, g \in (A^{-1})^m_n$ with $f^m = g^m$, and $T_m(1) = 1$. Since $T((A^{-1})^m_n) = (B^{-1})^m_n$, we have $T_m(((A^{-1})^m_n)^m) = ((B^{-1})^m_n)^m$. By (10), we also have

$$\left\| T_m(f^m) T_m(g^m)^{-1} - 1 \right\|_\infty = \left\| f^m (g^m)^{-1} - 1 \right\|_\infty$$

for all $f^m, g^m \in ((A^{-1})^m_n)^m$. Notice that $((A^{-1})^m_n)^m, ((B^{-1})^m_n)^m$ are subgroups that contain $\exp A, \exp B$ respectively. Proposition 7 shows that there exists a real-algebra isomorphism $\widetilde{T} : A \to B$, a clopen subset $\mathcal{K}$ of M_B and a homeomorphism $\Phi : M_B \to M_A$ such that

$$\widehat{\widetilde{T}(f)} = \begin{cases} \hat{f} \circ \Phi & \text{on } \mathcal{K} \\ \overline{\hat{f} \circ \Phi} & \text{on } M_B \setminus \mathcal{K} \end{cases} \tag{11}$$

for every $f \in A$ and $\widetilde{T}(f^m) = T_m(f^m)$ for every $f^m \in ((A^{-1})^m_n)^m$. By the definition of T_m and equation (11), we have $\widetilde{T}(f)^m = (T(f)/T(1))^m$ for every $f \in (A^{-1})^m_n$. By (10) and (11), we also have

$$\left\| \frac{(T(f)/T(1))^m}{(T(g)/T(1))^m} - 1 \right\|_\infty = \left\| \frac{T(f)^m}{T(g)^m} - 1 \right\|_\infty = \left\| \frac{f^m}{g^m} - 1 \right\|_\infty = \left\| \frac{\widetilde{T}(f)^m}{\widetilde{T}(g)^m} - 1 \right\|_\infty$$

for every $f \in S_A$ and $g \in (A^{-1})^m_n$. Since $(B^{-1})^m_n$ contains $\exp B$, we obtain

$$\|(T(f)/T(1))^m \mathfrak{g} - 1\|_\infty = \|\widetilde{T}(f)^m \mathfrak{g} - 1\|_\infty$$

for every $f \in S_A$ and all $\mathfrak{g} \in \exp B$. By Lemma 6, we obtain

$$\widetilde{T}(f)^m = (T(f)/T(1))^m \tag{12}$$

for every $f \in S_A$.

Finally, we will show that $\widetilde{T}(f)^d = (T(f)/T(1))^d$ for every $f \in S_A$, where d is the greatest common divisor of m and n. By raising both sides of equation (12) to the n-th power, we have $\widetilde{T}(f)^{mn} = (T(f)/T(1))^{mn}$ for every $f \in S_A$. By (11), we also have

$$\|\widetilde{T}(f)(T(g)/T(1))^{-mn} - 1\|_\infty = \left\|fg^{-mn} - 1\right\|_\infty \tag{13}$$

for every $f \in A$ and $g \in T^{-1}((B^{-1})^m_n)$. If we consider the map T_n on $((A^{-1})^n_m)^n = \{f^n : f \in (A^{-1})^n_m\}$ defined as $T_n(f^n) = T(f)^n/T(1)^n$ for $f^n \in ((A^{-1})^n_m)^n$, similar arguments show that there exists a real-algebra isomorphism $\widetilde{T}' : A \to B$ such that $\widetilde{T}'(f)^n = (T(f)/T(1))^n$ for every $f \in S_A$ and

$$\|\widetilde{T}'(f)(T(g)/T(1))^{-mn} - 1\|_\infty = \left\|fg^{-mn} - 1\right\|_\infty$$

for every $f \in A$ and $g \in T^{-1}((B^{-1})^n_m)$. Together with (13), this shows that

$$\|\widetilde{T}(f)(T(g)/T(1))^{-mn} - 1\|_\infty = \|\widetilde{T}'(f)(T(g)/T(1))^{-mn} - 1\|_\infty$$

for every $f \in A$ and $g \in T^{-1}((B^{-1})^m_n \cap (B^{-1})^n_m)$. Since $(B^{-1})^m_n$ and $(B^{-1})^n_m$ contain $\exp B$, we obtain

$$\|\widetilde{T}(f)\mathfrak{g} - 1\|_\infty = \|\widetilde{T}'(f)\mathfrak{g} - 1\|_\infty$$

for every $f \in A$ and all $\mathfrak{g} \in \exp B$. It follows from Lemma 6 that $\widetilde{T} = \widetilde{T}'$ on A. Consequently, $\widetilde{T}(f)^n = (T(f)/T(1))^n$ for every $f \in S_A$, which implies that $\widetilde{T}(f)^d = (T(f)/T(1))^d$ for every $f \in S_A$. □

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